JOURNAL OF PROPULSION AND POWER
Vol. 24, No. 5, September–October 2008

Technical Notes

TECHNICAL NOTES are short manuscripts describing new developments or important results of a preliminary nature. These Notes should not exceed 2500 words (where a figure or table counts as 200 words). Following informal review by the Editors, they may be published within a few months of the date of receipt. Style requirements are the same as for regular contributions (see inside back cover).

Ducted Wind/Water Turbines and Propellers Revisited

M. J. Werle* and W. M. Presz Jr.†
FloDesign Inc., Wilbraham, Massachusetts 01095

DOI: 10.2514/1.37134

Nomenclature

A = flow cross-sectional area
C_P = power coefficient [see Eq. (4c)]
C_p = pressure coefficient [see Eq. (8)]
C_S = duct/shroud force coefficient [see Eq. (2a)]
C_T = prop thrust coefficient [see Eq. (6)]
C_{TP} = total thrust coefficient based on velocity V_P [see Eq. (12)]
F_S = duct/shroud force
k = diffuser loss coefficient
P = power
p = pressure
r = $C_P/(16/27)$
T = thrust
V = velocity
V_c = characteristic velocity [see Eq. (10b)]
V_P = power velocity [see Eq. (10c)]
v = V/V_a
ρ = fluid density

Subscripts

a = ambient freestream conditions
D = properties at the duct exit plane
i = upstream flow inlet/capture area
max = maximum-power state
o = downstream outlet area
p = conditions at the propeller plane
T = total thrust
0 = properties when $V_a = 0$
1,2 = properties fore and aft of the prop, respectively

Introduction

THERE has been considerable effort and discussion in the literature concerning the potential for ducted wind/water turbines, as depicted in Fig. 1, to extract more power than their bare-propeller counterparts (i.e., to surpass the theoretical power-extraction limit defined by Betz for bare propellers over 80 years ago, as discussed in detail in most power or propulsion texts such as [1–5]). This limit of only 59.3% of the power in the freestream fluid flowing through the area swept by the bare propeller provides a difficult-to-attain upper limit for the performance one can expect of existing bare-propeller wind and water turbines. To date, few, if any, designs have exceeded the level of 50%. However, Igar [6,7], Riegler [8], Hansen et al. [9], and many others (see, for example, [10–12]) have presented results that clearly, but empirically, demonstrate the important and tantalizing possibility of exceeding this level through the addition of ducts/shrouds around the propeller. However, it has been found that, as discussed herein, the majority of these ducted-wind-turbine studies were based on an incomplete formulation of the problem that led to incorrect limits for their predicted potential performance gain over their bare-propeller counterparts. The simple but corrected formulation and results presented herein 1) introduce a unified momentum model, based on first principles, applicable to both bare and ducted/shrouded wind and water turbines; 2) show how to properly generalize the Betz limit for bare propellers to the ducted/shrouded propeller case and thereby extract higher power levels; 3) identify a single critical shroud aerodynamic parameter that controls the power-extraction level; 4) uncover the appropriate nondimensional scaling parameters; and 5) provide a firm basis for further development of ducted-wind/water-turbine technology.

Additionally, this new formulation provides some interesting new results, insights, and handy relations applicable to ducted/shrouded propeller propulsion systems.

Ducted/Shrouded Wind/Water Turbines

Figure 1 provides the geometry and nomenclature applied herein to formulate a standard momentum-balance-based model for inviscid incompressible flow through a propeller or turbine system as discussed, for example, in [1–5]. The governing equations are written for a control volume using a cut incorporating the turbine (modeled as an actuator-disc discontinuity with zero leakage around its edge) and the duct/shroud (with its attendant force on the flow, F_S), along with parallel constant static pressure inflows and outflows far upstream and downstream. In the current formulation, two changes over those of previous studies are introduced. First, although all previous momentum-balance-based formulations for bare wind/water turbines (and ducted propulsion propellers) have correctly imposed the constant pressure boundary condition far downstream, to date, this has not been done for ducted wind/water turbines. References [6–8], for example, incorrectly impose contrived pressure conditions at the duct exit, A_D, shown in Fig. 1. More details on this point will be provided subsequently. In the momentum-balance model and formulation developed herein, the freestream pressure boundary condition is imposed far downstream, as required for low-speed or incompressible flow. From this, a momentum balance for the system of Fig. 1 leads to the relation

$$A_p(p_{p2} - p_{p1}) + F_S = \rho A_p V_p (V_o - V_a) \tag{1a}$$

and use of Bernoulli's equation to relate the pressure p_{p1} to the upstream conditions and p_{p2} to the downstream conditions gives

$$p_{p2} - p_{p1} = \tfrac{1}{2}\rho\left(V_o^2 - V_a^2\right) \tag{1b}$$

Received 13 February 2008; revision received 30 April 2008; accepted for publication 30 April 2008.

*Chief Scientist, 380 Main Street. Fellow AIAA.
†Chief Technology Officer, 380 Main Street. Member AIAA.

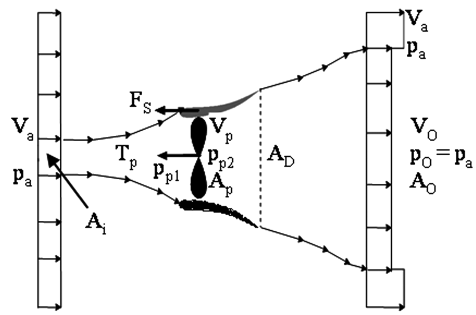

Fig. 1 Ducted-system nomenclature.

The pressure jump at the actuator disc, $p_{p2} - p_{p1}$, is then related to the power injected or extracted by the propeller through the relation

$$P = A_p V_p (p_{p2} - p_{p1}) = \tfrac{1}{2}\rho A_p V_p \left(V_o^2 - V_a^2\right) \tag{1c}$$

The second change introduced here is related to the duct/shroud induced force term F_S, appearing in Eq. (1) and shown in Fig. 1. To establish an independent relation for this term, an entirely new formulation is introduced based on classical fluid dynamic theory. As discussed in [1,13,14], for example, for the inviscid incompressible shrouded flow depicted in Fig. 1, any axial pressure change due to the energy addition or extraction by a propeller causes the flow streamlines to expand or contract laterally, giving rise to a velocity component normal to the shroud. Because of this, the Kutta–Joukowski theorem requires an axial force F_S to occur on the shroud as a result of the interaction between this induced velocity component normal to the shroud axis and the ring-vortex vector associated with the duct's/shroud's aerodynamic circulation. The critical aspect of this classical theoretical model is that it relates the axial force on the shroud solely and directly to the energy addition or extraction at the propeller location. Based on this classical theoretical foundation plus dimensional analysis considerations, the current formulation takes the shroud force F_S to be proportional to the force induced by the pressure change across the propeller, $A_p(p_{p2} - p_{p1})$, through a newly defined axial force coefficient C_S, such that

$$C_S \equiv F_S/[A_p(p_{p2} - p_{p1})] \tag{2a}$$

Using Eq. (1b) in Eq. (2a) leads to the sought-after independent relation for the shroud force F_S as

$$F_S = \tfrac{1}{2}\left[\rho A_p \left(V_o^2 - V_a^2\right)\right] C_S \tag{2b}$$

To the authors' knowledge, Eqs. (2a) and (2b) represents the first introduction to the power and propulsion fluid mechanics community of this approach for representing the force induced on a duct/shroud by a propeller. Because it is based on classical fluid dynamics theory, it stands on it own merits and should be considered accordingly. The following discussion will provide insights into the utility of this model.

Using Eqs. (1) and (2), the resulting internal velocity at the turbine disk and total thrust produced are then given as

$$V_p = \tfrac{1}{2}(1 + C_S)(V_o + V_a) \tag{3a}$$

$$T_T \equiv A_p(p_{p2} - p_{p1}) + F_S = (1 + C_S)P/V_p = 2P/(V_o + V_a) \tag{3b}$$

It is worthy of note that with $C_S = 0$, Eqs. (1–3) recover both the classical bare wind-turbine and propulsion cases as provided in most fluid texts, as in [1–5,14].

For the ducted wind turbine, following Betz and others (see [1–5], for example), it is straightforward to determine the maximum power that can be extracted from the stream by a ducted wind/water turbine by setting to zero the derivative of the power P with respect to the downstream velocity V_O. Using Eq. (2b) in Eq. (1), this then gives the maximum power that can be extracted as

$$P_{\max} = -\frac{16}{27}(1 + C_S)\left(\frac{1}{2}\rho A_p V_a^3\right) \tag{4a}$$

which is found to occur when

$$V_o = V_a/3 \tag{4b}$$

These results are usually expressed in terms of the power coefficient C_P, defined as

$$C_P \equiv -P/\left[\tfrac{1}{2}\rho A_p V_a^3\right] \tag{4c}$$

which, with Eq. (4a), gives

$$C_{P\max} = \frac{16}{27}[1 + C_S] \tag{4d}$$

For the case of $C_S = 0$, Eq. (4a) recovers the classical Betz power-extraction limit of $16/27$ as discussed, for example, in [1–5]. More important, it also provides a ducted/shrouded wind-turbine generalization of that limit that is based on classical inviscid fluid dynamic theory, including the imposition of the freestream pressure level at downstream infinity. As such, it is offered here as the corrected alternative to the limit forms proposed by Igar [6,7] and Reigler [8] that resulted from their improper imposition of a pressure condition at the shroud/duct exit. This point is discussed further in relation to Eq. (8). The result presented in Eq. (4d) has also recently been independently verified by Jamieson [15] using a completely different approach.

Figure 2 shows the resulting influence of the shroud/duct force coefficient C_S on the maximum-power coefficient $C_{P\max}$ and its attendant area and velocity ratios, as defined in Fig. 1, as

$$V_p/V_a = A_i/A_p = 2/3[1 + C_S] \tag{5a}$$

$$V_p/V_o = A_o/A_p = 2[1 + C_S] \tag{5b}$$

As an example for demonstrating the validity and utility of the current formulation, comparisons are provided here with the more comprehensive and complex viscous computational fluid dynamics (CFD) results of Hansen et al. [2,9] in which they present a range of the power-extraction levels for flow through an actuator disk simulating a pressure drop through wind turbines. Results were presented for both the bare-propeller case and for an aerodynamically contoured ducted case with an aggressive exit area ratio, $A_D/A_p = 1.86$. Figure 3 reproduces the results from [2,9], which were presented in terms of the thrust on the actuator disk defined as

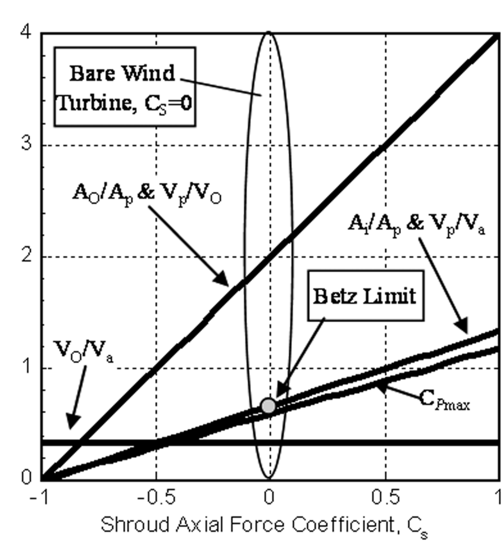

Fig. 2 Wind turbine at maximum power.

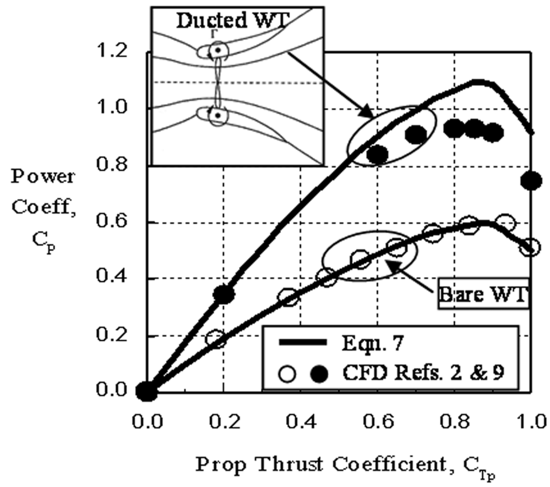

Fig. 3 Comparison with CFD analysis.

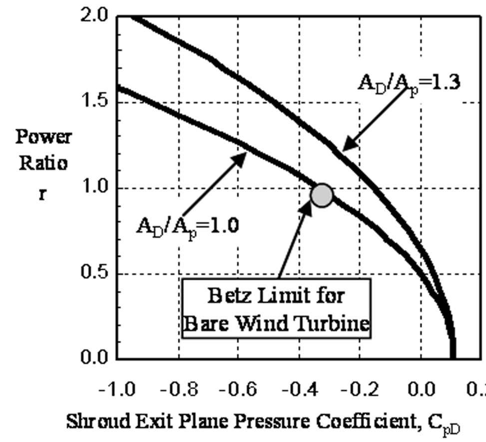

Fig. 4 Wind-turbine maximum-power limits.

$$C_T \equiv T_p / \left(\tfrac{1}{2}\rho A_p V_a^2\right) \tag{6}$$

where $T_{p,}$ was calculated from the pressure drop $p_{p2} - p_{p1}$ acting on the actuator disk area A_p.

For the current formulation, applying Eqs. (1–3) provides the relationship

$$C_P \equiv \tfrac{1}{2}(1 + C_S)C_T[1 + \sqrt{1 - C_T}] \tag{7}$$

Use of Eq. (7) requires determination, by independent means, of the duct/shroud force coefficient C_S. This can be done by first noting that the current formulation applies for all power-extraction levels, including that of the clear duct case with zero power extraction. Conveniently, Hansen et al. [9] did provide the flow parameters for this case. In particular, they gave that $V_p/V_a = 1.83$, which, when used in Eq. (3a) along with the fact that [from Eq. (1c)] $V_o = V_a$ when zero power is extracted (i.e., for a clear duct with no propeller present), thus leads to $C_S = 0.83$.

The resulting comparison presented in Fig. 3 shows that the current simple one-dimensional inviscid momentum-balance model agrees well with the more complex and comprehensive CFD results over the full range of the blade thrust for both the bare and ducted configurations. Not surprisingly, the viscous CFD results produce a slightly lower maximum-power level for the ducted case, due to the considerable viscous losses encountered for such an aggressive duct diffusion area ratio of $A_D/A_p = 1.86$.

To further relate the current formulation to earlier works (e.g., [6–8]), it is useful to first calculate the pressure level at the exit plane, A_D, of Fig. 1 using Eqs. (1–3) along with Bernoulli's equation to write the exit pressure coefficient as

$$C_{p_D} \equiv \frac{p_D - p_a}{\frac{1}{2}\rho V_a^2} = \left(\frac{V_O}{V_a}\right)^2 - \frac{(V_p/V_a)^2}{(A_p/A_D)^2} = \left(\frac{1 + C_S}{2}\right)^2 \left(1 + \frac{V_O}{V_a}\right)^2 \left(1 - k + k\left(\frac{A_p}{A_D}\right)^2\right) \tag{8}$$

where, as in [6–8], the diffuser static pressure-recovery-efficiency coefficient k has been introduced to relate the pressure rise from the blade/disk location to that at the exit plane, A_D.

In Eq. (8), to extract the maximum-power level, V_o/V_a and V_p/V_a must satisfy Eqs. (4b) and (5a), respectively. Thus, Eq. (8) dictates that the exit pressure coefficient C_{pD} and the duct-area diffusion level A_D/A_p cannot be employed as independent variables, but rather must always satisfy this relation to extract the maximum power possible.

With Eq. (8), it is also convenient to employ Igar's [6,7] definition of the ratio of the maximum power available to the Betz limit value for a bare propeller of the same diameter. This is defined as r, a measure of the power augmentation due to the duct/shroud, and is determined using Eqs. (1–6) to write that

$$r \equiv \frac{27}{16} C_{P\max} = \frac{1}{2}\sqrt{\frac{1 - 9C_{pD}}{1 - k + kA_p/A_D}} \tag{9}$$

This result differs significantly from, and necessarily supersedes, those proposed by Igar [7] and Riegler [8], who imposed pressure conditions at the duct/shroud exit that they incorrectly took to be independent of the power extracted, which violates the conditions dictated by Eq. (8).

Results from the use of Eq. (9) are shown in Fig. 4 for duct-area diffusion ratios $A_D/A_p = 1.0$ and 1.3 for a diffuser pressure-recovery coefficient $k = 1$. Ducts with area diffusion ratios much larger than 1.3 are found to suffer significant losses ($k < 1$, as was the case in [4]) unless they are quite long and thus too heavy to be practical. Figure 4 clearly shows that ducted wind/water turbines are theoretically capable of extracting power levels significantly above those of their bare-propeller counterparts for realistic levels of diffusion and exit pressure levels.

Ducted/Shrouded Propellers

As a final point, it is additionally noted that the formulation of Fig. 1 given in Eqs. (1–3) also applies to the ducted-propulsive-propeller case with known power input. For this case, Eqs. (1–3) can be manipulated to write

$$v_o^3 + v_a v_o^2 - v_a^2 v_o - 1 - v_a^3 = 0 \tag{10a}$$

where use has been made of the following definitions:

$$V_c \equiv (1/(1 + C_S))^{1/3} V_P \tag{10b}$$

$$V_P \equiv (4P/\rho A_p)^{1/3} \tag{10c}$$

$$v_o \equiv V_o/V_c \tag{10d}$$

$$v_a \equiv V_a/V_c \tag{10e}$$

Note that the power velocity V_P introduced in Eq. (10c) is closely related to the disk-loading coefficient used by others (e.g., [14]).

The solution to the cubic Eq. (10a) can be expressed in closed form, which in turn can be approximated using a Taylor series as

$$v_o \approx 1 - \tfrac{1}{3}v_a + \tfrac{4}{9}v_a^2 \tag{11}$$

Equation (11) can then be used in Eq. (3b) to calculate the ducted system's total thrust in terms of a thrust coefficient, defined as

$$C_{TP} \equiv \frac{T_T}{\frac{1}{2}\rho A_p V_P^2} = \frac{(1 + C_S)^{1/3}}{v_a + v_o} \approx \frac{(1 + C_S)^{1/3}}{1 + \frac{2}{3}v_a + \frac{4}{9}v_a^2} \tag{12}$$

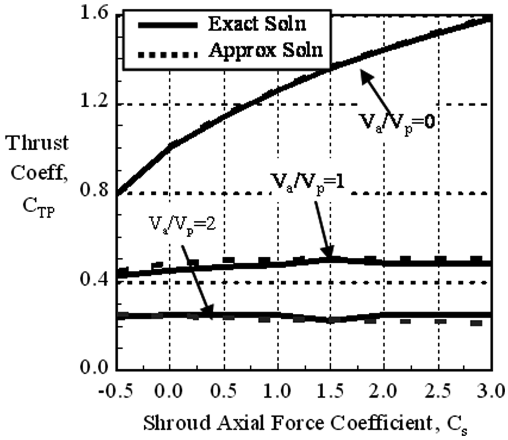

Fig. 5 Ducted-prop performance.

As shown in Fig. 5, the static flight or hover case ($v_a = 0$) for the bare propeller ($C_{TP} = 1$) is recovered at $C_S = 0$ as only one of an infinite family of cases, all of which are well represented by the simple polynomial approximation form of Eq. (12) for both static and forward-flight conditions.

The current formulation can be further simplified by first noting that for the static case, $V_a = 0$, and the duct exit-plane pressure coefficient of Eq. (8) with V_P replacing V_a can be used with the approximate form of Eq. (12) and further Taylor series approximations to determine the static flight or hover ($V_a = 0$) thrust level as

$$C_{TP0} \equiv (C_{TP})_{V_a=0} = (1 + C_S)^{1/3} \approx (2A_D/A_p)^{1/3} - (A_D/A_p)C_{pP_D} \quad (13)$$

For current applications, it has been further assumed that the diffuser efficiency k is unity. The resulting Fig. 6 shows that for the static case,

1) A thrust increase of nearly 80% above the bare-propeller level is attainable with moderate diffusion and exit-plane suction pressures.

2) The handy approximation of Eq. (12) leads to a good representation over the regimes of interest.

Finally, combining Eqs. (12) and (13) leads to a simple relationship for forward-flight effects on the thrust as

$$T_T/T_{T_0} = C_{TP}/C_{TP_0} \approx 1/[1 + (\tfrac{2}{3}C_{TP_0}V_a/V_P) + (\tfrac{2}{3}C_{TP_0}V_a/V_P)^2] \quad (14)$$

which is shown in Fig. 7 to yield a very accurate approximation to the exact solution for virtually all values of forward velocity. Further demonstration of the validity and utility of the preceding formulation for propulsive propeller systems is provided in [16].

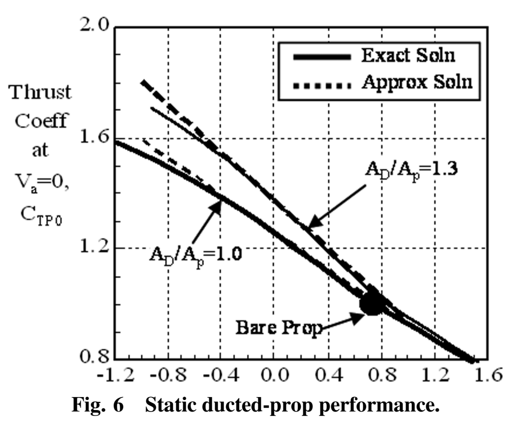

Fig. 6 Static ducted-prop performance.

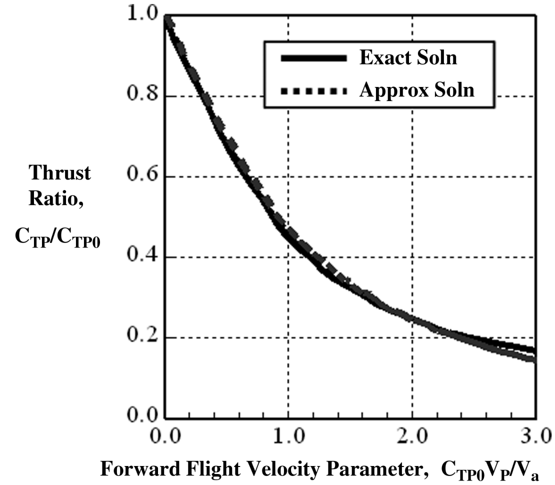

Fig. 7 Ducted-prop velocity effect.

Conclusions

A new, simple, and unified analytical model for representing ducted/shrouded propeller-based propulsion and power systems is now at hand. Because it is based on classical fluid dynamic theories without empirical elements, it stands on its own merits.

From the closed-form, algebraically simple, analytical predictions presented herein, it can be concluded that

1) Ducted turbines are theoretically capable of extracting significantly more power than a bare wind/water turbine.

2) There is but a single parameter, the duct/shroud nondimensional force coefficient that determines the maximum power extractable.

3) One can straightforwardly predict the ducted-wind/water-turbine maximum attainable performance based on the flow characteristics of the empty or clear duct configuration.

4) With this new model in hand, a rational approach to the design of ducted or shrouded wind/water turbines can precede with the potential for achieving the predicted maximum-power output. Without it, all previous such designs must necessarily be considered to be potentially suboptimal.

5) For ducted-propeller propulsion systems, for the first time, there exists a series of simple and handy algebraic relations for predicting performance, correlating data, and/or guiding preliminary design efforts with or without forward-flight effects.

References

[1] Janna, W., *Introduction to Fluid Mechanics*, 3rd ed., PWS Publishing, Boston, 1993.
[2] Hansen, M. O. L., *Aerodynamics of Wind Turbines*, Cromwell Press, Trowbridge, England, U.K., 2000.
[3] Gashe, R., and Twele, J., *Wind Power Plants*, Solarpraxis, Berlin, 2002.
[4] Hau, E., *Wind Turbines*, 2nd ed., Springer, New York, 2005.
[5] Kirke, B., "Developments in Ducted Water Current Turbines," http://www.cyberiad.net/library/pdf/bk_tidal_paper25apr06.pdf, 2006.
[6] Igar, O., "Shrouds for Aerogenerators," *AIAA Journal*, Vol. 14, No. 10, Oct. 1976, pp. 1481–83; also AIAA Paper 76-181 Jan. 1976.
[7] Igar, O., "Research and Development for Shrouded Wind Turbines," *Energy Conversion and Management*, Vol. 21, No. 1, 1981, pp. 13–48. doi:10.1016/0196-8904(81)90005-4
[8] Riegler, G., "Principles of Energy Extraction from a freestream by Means of Wind Turbines," *Wind Engineering*, Vol. 7, Feb. 1983, pp. 115–126.
[9] Hansen, M. O. L., Sorensen, N. N., and Flay, R. G. J., "Effect of Placing a Diffuser Around a Wind Turbine," *Wind Engineering*, Vol. 3, No. 4, 2000, pp. 207–213. doi:10.1002/we.37
[10] Lilley, G. M., and Rainbird, W. J., "A Preliminary Report on the Design and Performance of a Ducted Windmill," College of Aeronautics, Rept. 102, Cranfield, England, U.K., 1956.
[11] Toucher, A. J., "Habitat Friendly, Pressure Conversion, Wind Energy Extraction," U.S. Patent 6,887,031 B1, Issued 3 May 2005.

[12] Gaskell, C., "Ducted Wind Turbine," U.S. Patent 7,018,166 B2, Issued 28 Mar. 2006.
[13] McCormick, B. W., and Eisenhuth, J. J., "Design and Performance of Propellers and Pump Jets for Underwater Propulsion," *AIAA Journal*, Vol. 1, No. 10, 1963, pp. 2345–2352.
[14] McCormick, Barnes, W., Jr., *Aerodynamics of V/STOL Flight*, Dover, Mineola, NY, 1999.
[15] Jamieson, J., "Beating Betz—Energy Extraction Limits in a Uniform Flow Field," European Wind Energy Conference, European Wind Energy Association Paper CS3.2Apr. 2008.
[16] Werle, M. J., and Presz, W. M., Jr., "New Developments in Shrouds and Augmentors for Subsonic Propulsion Systems," AIAA Joint Propulsion Conference, AIAA Paper 2008-4962, 2008.

A. Prasad
Associate Editor